TASI: Cosmology V

Full set of Exercises at `http://home.fnal.gov/~dodelson/TASI`

Definitions

1. Overdensity: $\delta \equiv (\rho - \bar{\rho})/\bar{\rho}$
2. Wavelength which enters the horizon at equality, $k_{\rm eq} = 0.073\Omega_m h^2$ Mpc^{-1}
3. Power spectrum of density fluctuations

$$\langle \tilde{\delta}(\vec{k})\tilde{\delta}(\vec{k}')\rangle = (2\pi)^3 \delta^3(\vec{k} + \vec{k}') P_\delta(k).$$

Equations

1. Poisson's Equation in Fourier space

$$k^2 \tilde{\Phi} = \frac{4\pi G \rho_m a^2}{k^2} \tilde{\delta}.$$

2. Evolution equation (sub-horizon, no radiation)

$$\frac{d^2\delta}{da^2} + \left(\frac{d\ln(H)}{da} + \frac{3}{a}\right)\frac{d\delta}{da} - \frac{3\Omega_m}{2a^5}\frac{H_0^2}{H^2}\delta = 0.$$

3. Images are sheared by intervening matter. The shear is decsribed by a 2x2 matrix with elements

$$\begin{pmatrix} -\kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & -\kappa + \gamma_1 \end{pmatrix} = 2\int_0^\chi d\chi' \frac{\partial^2 \Phi(\vec{x}(\chi'))}{\partial x^i \partial x^j} \chi'(1 - \chi'/\chi).$$

Here κ is the convergence which governs the magnification of an image and γ_1 and γ_2 are the two components of shear. The potential is evaluated at 3D position $\vec{x} \simeq \chi'(\theta_x, \theta_y, 1)$ assuming the line of sight is close to the z-axis. The shear matrix then depends on the angular position on the sky and can be Fourier tranformed or spherical harmonic-ed, just as are the anisotropies in the CMB. The power spectrum of both $\tilde{\gamma}_i$'s are then related to the power spectrum of the convergence, which in turn can be expressed as an integral along the line of sight of the 3D gravitational potential power spectrum:

$$P_\kappa(l) = \frac{l^4}{4}\int_0^\infty d\chi \frac{g^2(\chi)}{\chi^6} P_\Phi(l/\chi).$$

Here $g = 2\chi(1 - \chi/\chi_s)$ for sources at distance χ_s.

Exercises

1. Solve the evolution equation when $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. To obtain the growing mode, define $u = \delta/H$. Rewrite the evolution equation in terms of u and integrate. You should get an expression for the growth function as an integral over a function of the Hubble rate.
2. One popular way to characterize power on a particular scale is to compute the expected RMS overdensity in a sphere of radius R,

$$\sigma_R^2 \equiv \langle \delta_R^2(x) \rangle.$$

Here

$$\delta_R(\vec{x}) \equiv \int d^3x' \, \delta(\vec{x}') W_R(\vec{x} - \vec{x}')$$

where $W_R(x)$ is the *tophat* window function, equal to 1 for $x < R$ and 0 otherwise; the angular brackets denote the average over all space. By Fourier transforming, express σ_R in terms of an integral over the power spectrum.